A mixed integer programming approach to the economic load dispatch with valve-point effect and transmission losses

Augustin Lefèvre, Easter Selvan, P-A Absil

Mathematical Engineering Department

UCL Université catholique de Louvain

n power units, producing p_i MW.

Goal : minimize production subject to fixed demand

$$\min_p \sum_{i=1}^n \overbrace{a_i + b_i p_i + c_i^2 p_i + \underbrace{d_i |\sin(e_i(p_i - p_i^{\min})|}_{\text{valve-point effect}}}^{f_i(p_i)} \quad (1)$$

$$\text{subject to} \quad p^{\min} \leq p \leq p^{\max} \quad (2)$$

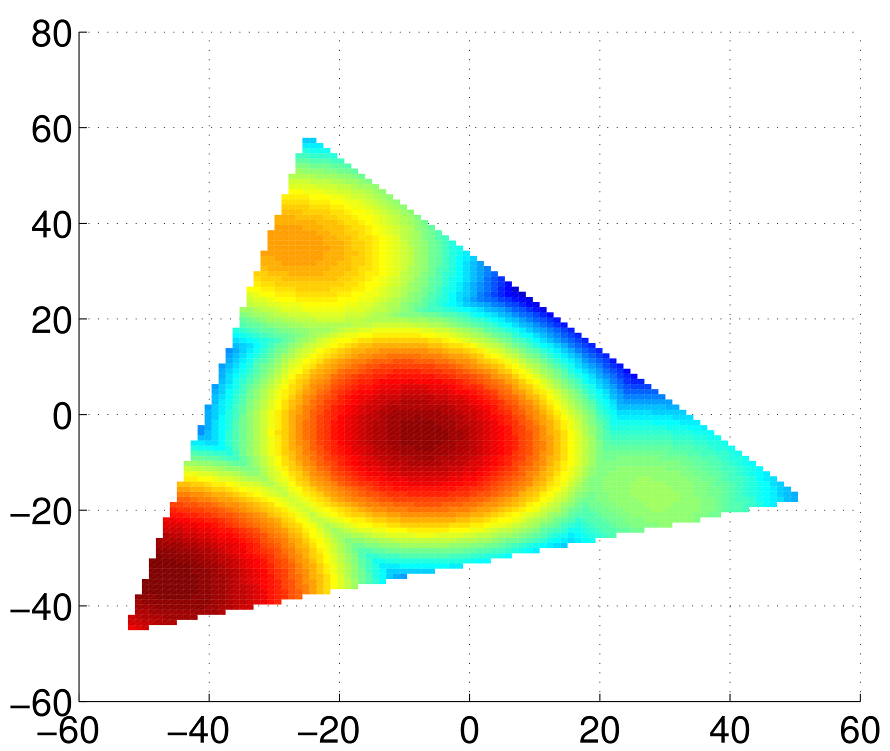
$$p^\top B p + p^\top B_0 + B_{00} + p_{\text{load}} \leq \sum_{i=1}^n p_i \quad (3)$$

Quadratic transmission losses : $B \succeq 0$, $B_0 \geq 0$, $B_{00} \geq 0$

We assume B, B_0 small enough so at an optimum, equality holds in (3).

Nonconvex problem, so potentially many global optima

Idea : local minima can be found by local optimization. The global minimum can be found with a branch-and-bound heuristic.



Piecewise quadratic model of the objective function :

$$\tilde{f}_{i,s}(p_{i,s}) = \tilde{a}_{i,s} + \tilde{b}_{i,s} p_{i,s} + \tilde{c}_{i,s} p_{i,s}^2 \qquad P_{i,s} \leq p_{i,s} \leq P_{i,s+1}.$$

Split points are initialized as :

$$\begin{cases} P_{i,s} = p_i^{\min} + (s-1)\frac{\pi}{2e_i} \text{ if } i = 1 \dots S_i \\ P_{i,S_i+1} = p_i^{\max} \end{cases}$$

Unit i is divided in S_i segments.

Each segment has its own $p_{i,s}$ but they must be linked together.

MIQP formulation : $p_{i,s}$, i unit, s segment.

$$\min_{p,x} \quad \sum_{i=1}^n \sum_{s=1}^{S_i} \tilde{f}_{i,s}(p_{i,s}) \quad (4)$$

$$\text{subject to} \quad x_{i,s} P_{i,s-1} \leq p_{i,s} \leq x_{i,s} P_{i,s} \quad (5)$$

$$p^\top Q p + q^\top p + q_0 \leq 0 \quad (6)$$

$$x_{i,s} \in \{0,1\} \quad (7)$$

$$\sum_{s=1}^{S_i} x_{i,s} = 1 \quad (8)$$

$Q_{(i,s,(j,s')} = B_{ij}$, $q_{i,s} = -1 + B_{0,i}$, $q_0 = p_{\text{load}} + B_{00}$.

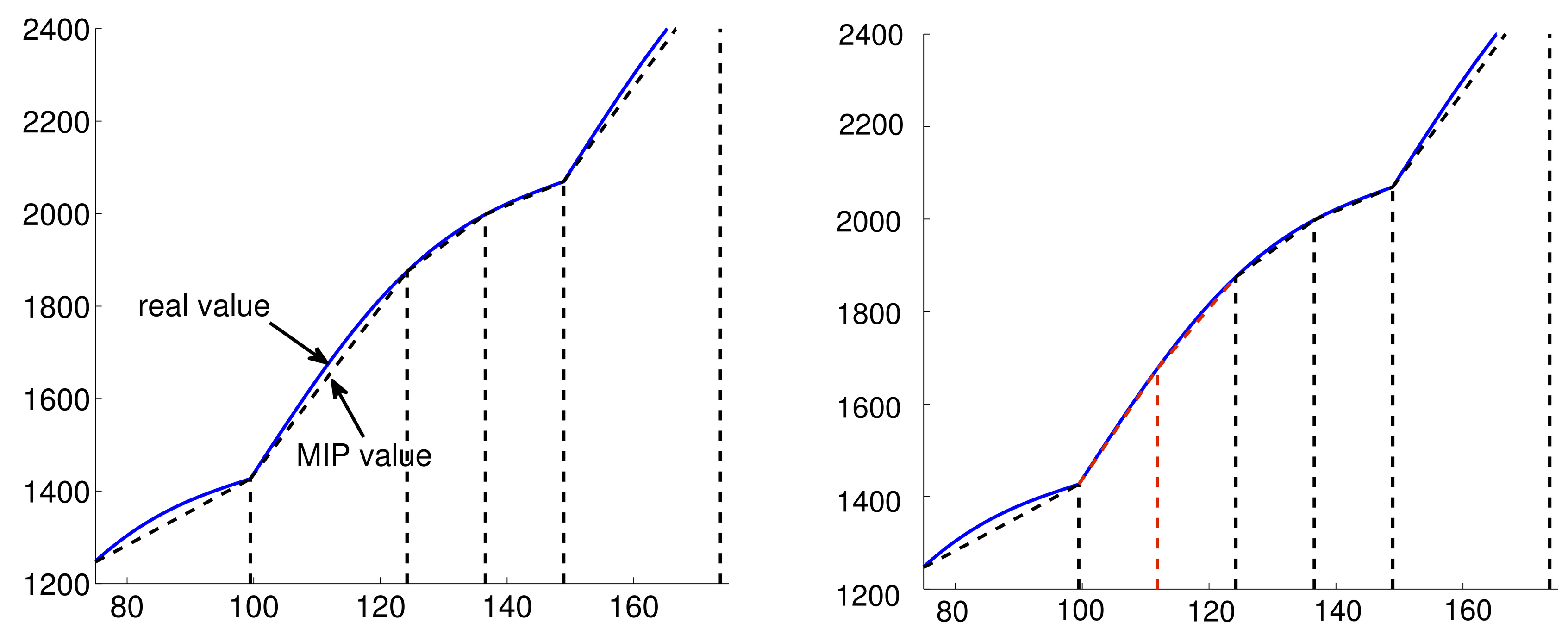
If $x_{i,s} = 1$, box constraints hold $P_{i,s} \leq p_{i,s} \leq P_{i,s+1}$.

(8) One and only one segment selected per unit.

Optimizing p for fixed x is easy (convex problem).

We can solve for x with a branch-and-bound technique.

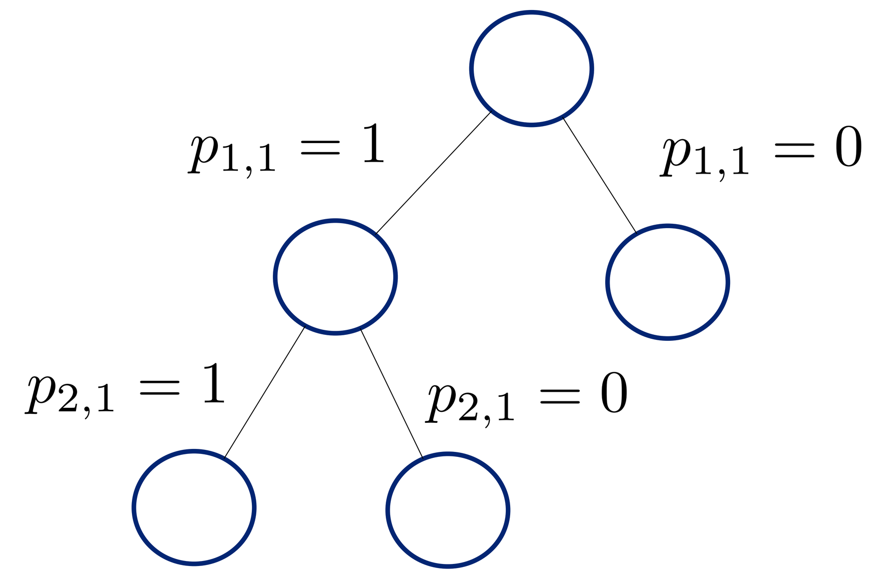
Solutions of the MIQP would be in one-to-one correspondence with ELDP...



... but the objective cost function is only approximate.

Under-approximation so $f^{\text{MIP}} \leq f^{\text{ELDP}}$.

Add solution p^* to split points, so $f(p^*_{i,s}) = \tilde{f}(p^*_{i,s})$.



Branch-and-bound techniques enumerate all possible solutions x efficiently by relying on bounds on the objective function of the problem : $l \leq f^{\text{MIP}} \leq m$.

Lower bounds at a given node are obtained by relaxing the $x_{i,s} \in \{0,1\}$ constraints to continuous constraints $x_{i,s} \in [0,1]$.

Upper bounds are obtained with feasible solutions.

We use Gurobi to solve the MIQP, with a gap of 1% (10% for $n = 40$).

Compared to heuristics such as Differential Evolution, MIQP retrieves better solutions, much more quickly.

n	DE	MIP (runtime)
3	3,199.01	3,199.01 (0.60)
5	834.13	834.13 (0.18)
6	930.46	925.41 (0.18)
15	33,772.14 (26.59)	33,700.11 (1.04)
40	122,624.35 (1167.35)	121,414.62 (96.71)

Figure 1: Fuel cost values for various data sets, obtained by the genetic algorithm (BSA), and the proposed MIP. n is the number of generating units.

Drawbacks : despite efficient heuristics, MIQP still suffer from combinatorial explosion.